STABILITY OF THE FREE-CONVECTION BOUNDARY LAYER ON A VERTICAL PERMEABLE PLATE

V. M. Eroshenko, L. I. Zaichik, and V. A. Pershukov

UDC 536.25:536.526

The hydrodynamic stability of flow in a free-convection boundary layer on a permeable plate is numerically investigated. The effect of the plate nonisothermicity and the injection blowing intensity on the critical parameters is determined.

The transition of laminar into turbulent flow in free convection on a vertical permeable isothermic surface was investigated experimentally in [1]. It was found that injection blowing destabilizes the flow, i.e., it reduces the flow stability with respect to external disturbances and the extent of the laminar boundary layer. We shall investigate the stability with respect to small disturbances in free-convection flow on impermeable and permeable vertical plates for different longitudinal gradients of the surface temperature.

1. Consider a self-similar boundary layer on a nonisothermic vertical permeable plate for power-law variations of the temperature drop $\Delta T = T_w - T_e \sim x^n$ and the injection velocity $v \sim x^{(n-1)/4}$. In this case, the equations of motion and energy are transformed into ordinary differential equations:

$$\begin{gathered} f''' + (n+3) ff'' - 2(n+1) f'^2 + \theta = 0, \\ \theta'' - \Pr[4nf'\theta - (n+3) f\theta'] = 0 \end{gathered} \tag{1}$$

with the boundary conditions

$$\eta = 0 \quad f = -A,\ f' = 0,\ \theta = 1;\ \eta = \infty \quad f' = 0,\ \theta = 0. \tag{2}$$

The stability of a free-convection boundary layer relative to small disturbances, with an allowance for the flow nonparallelism [2], is described by the following equations in terms of self-similar variables:

$$\begin{gathered} \varphi^{\mathrm{IV}} - 2\alpha^2\varphi'' + \alpha^4\varphi = i\alpha G(f' - C)(\varphi'' - \alpha^2\varphi) + \\ + ((3n+1) f'' + (n-1)\eta f''')\varphi' - ((n+3) f + (n-1)\eta f')(\varphi''' - \alpha^2\varphi') - i\alpha G f'''\varphi - \vartheta', \end{gathered} \tag{3}$$

$$\frac{1}{\Pr}(\vartheta'' - \alpha^2\vartheta) = i\alpha G(f' - C)\vartheta + (4n\theta + (n-1)\eta\theta')\varphi' - i\alpha G\varphi\theta' - ((n+3) f + (n-1)\eta f')\vartheta, \tag{4}$$

where the values of $4\nu(\mathrm{Gr}_x/4)^{1/2}/x$, and $x/(\mathrm{Gr}_x/4)^{1/4}$ are used as the velocity and the length scales, respectively.

The velocity and temperature fluctuations are assumed to be nonexistent at the wall, and exponential damping of disturbances is assumed at infinity. The boundary conditions for Eqs. (3) and (4) then have the form

$$\begin{gathered} \eta = 0 \quad \varphi = \varphi' = \vartheta = 0, \\ \eta = \infty \quad \varphi'' - \alpha^2\varphi = 0,\ \varphi' + \alpha\varphi = 0,\ \vartheta' + \alpha\vartheta = 0. \end{gathered} \tag{5}$$

G. M. Krzhizhanovskii Scientific-Research Power-Engineering Institute, Moscow. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 48, No. 3, pp. 382-387, March, 1985. Original article submitted February 3, 1984.

0022-0841/85/4803-0271$09.50

The eigenvalue problem (3)-(5) with an allowance for (1) and (2) for neutral disturbances $C_i = 0$ was solved by means of the method of differential trial runs presented in [3]. The calculations were performed for air flow, Pr = 0.73, with and without consideration of the effect of temperature fluctuations on velocity fluctuations that is accounted for by the last term in Eq. (3).

2. Consider free-convection flow on an isothermic permeable plate (n = 0). In this case, the solution of Eq. (1) with the boundary conditions (2) for the $0 \leqslant A \leqslant 1$ range of injection parameter values has been obtained by several authors, for instance in [4, 5]. It is evident from Fig. 1a, which shows the velocity and temperature profiles, that, as A becomes larger, the boundary layer thickness increases, and the fullness of the velocity profile decreases, while the temperature distribution tends to assume a bar form.

Figure 2 shows the neutral curves for different values of the parameter A. In the absence of injection, the obtained neutral curves show good correlation with data from [6]. However, there is no complete agreement between the results because the data from [6], in contrast to the data from our analysis, have been obtained by means of an equation not accounting for the transverse velocity component. It is evident from the figure that the neutral curve calculated on the basis of the complete system of equations (3) and (4) has two minimums for the zero or small injection parameter values. One of them, which characterizes the global flow stability, is determined by the interaction between thermal and hydrodynamic disturbances and lies in the longwave range of the wave number spectrum (small α values). As the injection intensity increases, the flow stability diminishes, i.e., the Grashof critical number (which is the analog of the critical Reynolds number for forced convection) decreases, which is in qualittative agreement with the experimental data from [1], obtained in determining the critical Grashof number for the start of transition, which is defined with respect to the development of irregular disturbances. The destabilizing effect of injection blowing on the flow is mainly connected with the distortion of the axial velocity distribution of undisturbed flow, which consists in the fact that the boundary layer thickness increases, while the velocity profile becomes less full.

For A = 0.34, both minimums of the neutral curve assume equal values, which leads to discontinuities in the relationships $\alpha_{cr}(A)$, $C_{r_{cr}}(A)$ and a break in $G_{cr}(A)$ (Fig. 3a). If the injection parameter increases still further, the short-wave disturbances (large α values) become the most unstable ones, while the neutral curves have a single minimum, similar to those calculated without an allowance for temperature fluctuations. Figures 2 and 3a indicate that, under these conditions, the solid and dashed curves converge, i.e., the effect of temperature fluctuations on flow stability diminishes. In the case of intensive injection, as was shown in [7], the entire flow region in the boundary layer can be divided into two zones: the inner (wall) zone, where the flow is calculated in the nonviscous approximation, and the outer zone (mixing layer), where the velocity and temperature distributions of the inner zone join those of the region remote from the plate. In Fig. 1a, the dash-dot curves represent the velocity and temperature profiles for the injection parameter A = 2, which are calculated by solving the motion and energy equations without considering the viscosity and thermal conductivity:

$$\eta/A = \sqrt{2}\int_{-1}^{f/A} \frac{dz}{\sqrt{|z|^{\frac{4n}{n+3}}\left(1-|z|^{\frac{4}{n+3}}\right)}}, \quad \theta = |f/A|^{\frac{4n}{n+3}}. \tag{6}$$

It is evident from the diagram that the velocity and temperature distributions (6) are in satisfactory agreement with the complete solutions of the system of equations (1) in the wall region. In the inner zone, regardless of the fact that it does not contain an inflection point, the stability loss mechanism is nonviscous in character, i.e., the stability parameters can be determined by solving the problem of nonviscous instability of forced flow for intensive injection with a negative pressure gradient $\beta = 2/3$, while they indicate an increase in the critical Grashof number [8]. However, in calculations based on Eqs. (3) and (4), the critical number G_{cr} is constant in the case of intensive injection (Fig. 3a), which is explained by the development of velocity perturbations of greater instability in the outer region, which has finite dimensions for n = 0, while the mathematical description of the laminar flow is given by

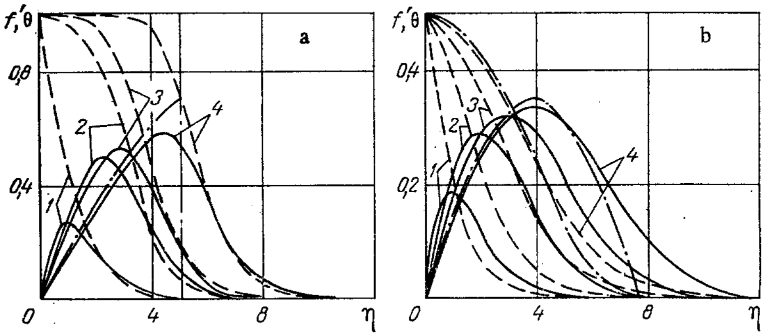

Fig. 1. Velocity (solid curves) and temperature (dashed curves) distributions in the boundary layer for a) n = 0 and b) n = 1. 1) A = 0; 2) 1; 3) 1.5; 4) 2.

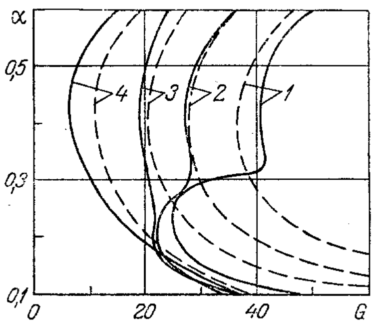

Fig. 2. Neutral stability curves, calculated with (solid curves) and without (dashed curves) an allowance for temperature fluctuations. 1) A = 0; 2) 0.2; 3) 0.4; 4) 1.0.

$$f''' + 3f''f - 2f'^2 + \theta = 0,$$
$$\theta'' + 3\Pr f\theta' = 0, \tag{7}$$
$$\eta_1 \to -\infty \quad f = \eta_1/\sqrt{2};\ f' = 1/\sqrt{2},\ \theta = 1;\ \eta_1 \to \infty \quad f' = 0,\ \theta = 0,$$

where $\eta_1 = \eta - \eta_0$, and $\eta_0 = \frac{3\sqrt{2}}{4} B(1/2, 3/4)$ is the position of the separating streamline.

Moreover, in the case of intensive injection, the outer critical point, where, as was shown in [9], the transition occurs, as well as the inflection points of the velocity and the temperature profiles, are located in the mixing layer, which actually determines the flow stability as a whole. In the case of intensive injection, as in forced convection [8], the value of α_{cr} diminishes with an increase in A (Fig. 3a), i.e., long-wave disturbances predominate among the disturbances developing in the flow.

It should be mentioned that, on the basis of the local similarity principle, we reach the conclusion that the above results can be used to analyze the effect on stability of not only self-similar, but also uniform, injection (as well as injection obeying some other law not greatly different from the self-similar injection law $v \sim x^{-1/4}$); in this case, the results must be compared for the same value of the injection parameter $vxCr_x^{-1/4}/\nu$. The above assumption is also supported by the fact that the flow in the outer zone of the boundary layer, which determines the loss of stability in the case of intensive injection, is described by problem (7) for both self-similar and uniform injection [10].

3. Consider nonisothermic flow on a vertical plate (n > 0). The velocity and temperature profiles for different values of n were investigated in [11]. It was found that the fullness

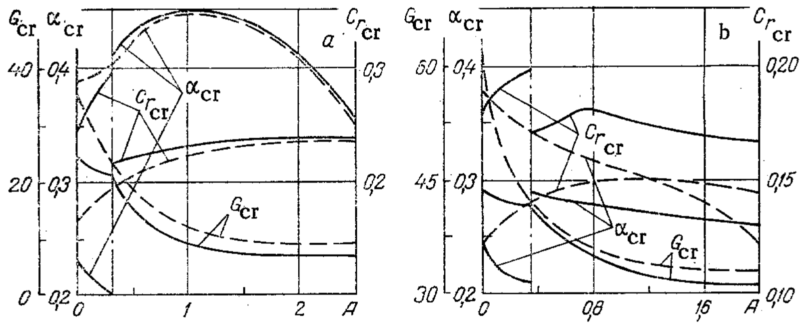

Fig. 3. Effect of injection on the critical stability parameters for a) n = 0 and b) n = 1.

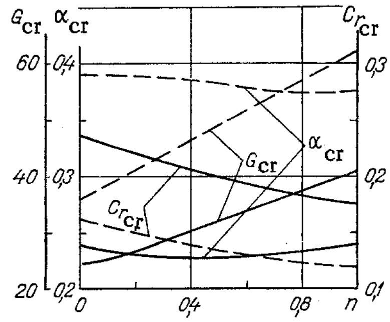

Fig. 4. Effect of nonisothermicity of the plate on the stability characteristics.

of the velocity profile increases, so that friction at the wall increases. Such behavior of the velocity profile enhances the flow stability, which can be seen in Fig. 4, where the dependences of the critical parameters on n are given. It is evident that α_{cr} is virtually constant, while Cr_{cr} diminishes with an increase in n, which corresponds to the behavior of the velocity maximum for undisturbed flow. We should also mention the virtually linear relationship $G_{cr}(n)$, which holds for calculations with or without an allowance for temperature fluctuations.

In order to determine the effect of transverse flow of matter on the flow stability near a nonisothermic surface, we shall consider the case n = 1, which corresponds to linear temperature variation along the plate. The velocity and temperature distributions determined by solving system (1) for n = 1 are shown in Fig. 1b. As the injection intensity rises, the boundary layer thickness increases, while the velocity profile approaches the sinusoidal form (dash—dot curves) obtained from (6) for n = 1 and A = 2.0.

The neutral curves calculated for this case have the same shape as for n = 0. Figure 3b shows the critical stability characteristics as functions of the injection parameter. As the intensity of the transverse mass flow increases, the Grashof number decreases, while the instability of the free-convection boundary layer is determined by the rise of long-wave disturbances for low injection values, as in the case of n = 0. For A > 0.34 (as for n = 0), short-wave disturbances are the most unstable ones. It is seen in Fig. 3b that, for intensive injection, the solid and the dashed curves draw nearer to each other, i.e., the effect of temperature fluctuations on the stability characteristics diminishes. On the whole, the effect of injection on the stability of a free-convection boundary layer has the same character for both isothermic and nonisothermic plates.

NOTATION

x, y, Cartesian coordinates; T, temperature; v, injection velocity; ν, kinematic viscosity coefficient; ρ, density; β_ρ, thermal expansion coefficient; a, thermal diffusivity coefficient; g, acceleration due to gravity; $Gr_x = g\beta_\rho(T_w - T_e)x^3/\nu^2$, Grashof number; $Pr = \nu/a$, Prandtl number; $\eta = y(Gr_x/4)^{-1/4}/x$, self-similar coordinate; f, self-similar stream function; $\theta = (T - T_e)/(T_w - T_e)$, dimensionless temperature; $A = vx(Gr_x/4)^{-1/4}/(n + 3)\nu$, injection parameter; φ and ϑ, amplitudes of the velocity and temperature disturbances, respectively; α and $C = C_r + iC_i$, wave number and the phase velocity of disturbance propagation, respectively; $G = 4(Gr_x/4)^{1/4}$. Subscripts: e, external medium; w, wall; cr, critical.

LITERATURE CITED

1. P. I. Brdlik and A. V. Mochalov, "Experimental investigation of free convection with transpiration injection and suction on a vertical surface," Inzh.-Fiz. Zh., 10, No. 1, 3-10 (1966).
2. Haaland and Sparrow, "Analysis of boundary layer and jet stability in a gravitational field with an allowance for nonparallelism of the basic flow," Teploperedacha, No. 3, 7-14 (1973).
3. M. A. Gol'dshtik and V. N. Shtern, Hydrodynamic Stability and Turbulence [in Russian], Nauka, Novosibirsk (1977).
4. R. Eihhorn, "The effect of mass transfer of free convection," Trans. ASME, J. Heat Transfer, 82, 260-263 (1960).
5. J. F. Clarke, "Transpiration and natural convection: the vertical flat-plate problem," J. Fluid Mech., 57, pt. 2, 45-61 (1973).
6. C. A. Hieber and B. Gebhart, "Stability of vertical natural convection boundary layers: some numerical solutions," J. Fluid Mech., 48, Pt. 4, 625-646 (1971).
7. V. M. Eroshenko, L. I. Zaichik, and V. A. Pershukov, "Free laminar convection on a vertical nonisothermic plate with high-intensity injection," Inzh.-Fiz. Zh., 43, No. 6, 909-913 (1982).
8. V. M. Eroshenko, L. I. Zaichik, and V. A. Pershukov, "Calculation of the stability characteristics of boundary layers with injection in the presence of a negative pressure gradient," Izv. Akad. Nauk SSSR, Mekh. Zhidk. Gaza, No. 2, 60-64 (1984).
9. A. A. Szewczyk, "Stability and transition of the free-convection layer along a vertical plane plate," Int. J. Heat Mass Transfer, 5, No. 10, 903-914 (1962).
10. J. H. Merkin, "Free convection with blowing and suction," Int. J. Heat Mass Transfer, 15, No. 5, 989-999 (1972).
11. E. M. Sparrow and J. L. Greeg, "Similar solutions for free convection from a nonisothermal vertical plate," Trans. ASME, 80, 379-386 (1958).